Transverse Charge Densities
of
N* Resonances

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traditional method to obtain charge densities

e.g. for nuclei A>>1

by Fourier Transformation

of form factors in the Breit-Frame

$$\rho(\vec{r}) = \int \frac{d^3Q}{(2\pi)^3} e^{-i\vec{Q}\cdot\vec{r}} F(Q^2)$$

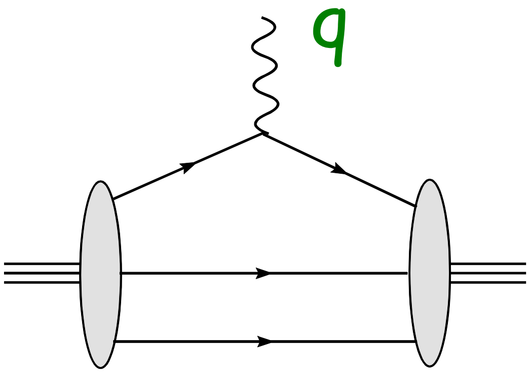
$$= \int_0^\infty \frac{dQ}{2\pi^2} Q^2 j_0(Q r) F(Q^2)$$

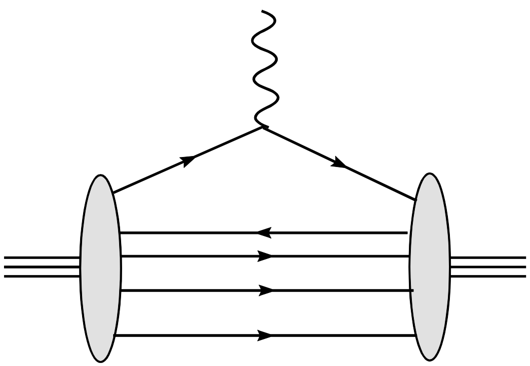
spherical
charge density

spherical
Bessel function

form factor
in the Breit frame

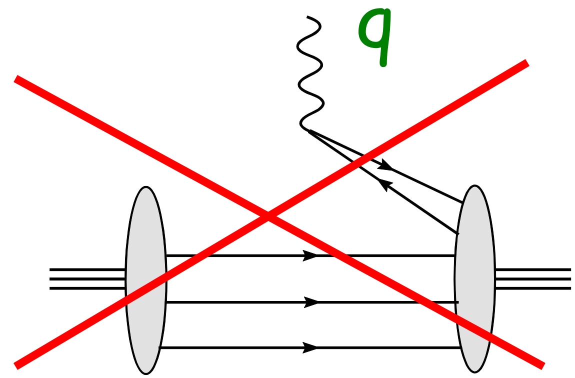
interpretation of FF as quark density





overlap of wave function Fock components with same number of quarks

interpretation as probability/charge density



overlap of wave function Fock components with different number of constituents

NO probability/charge density interpretation

absent in a LIGHT-FRONT frame !

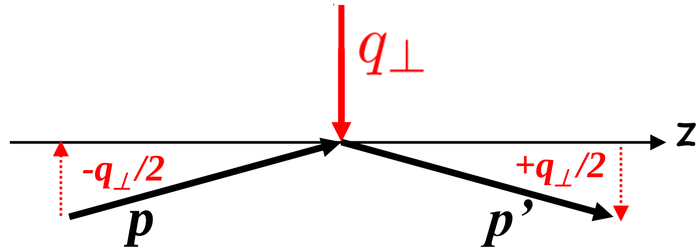
$$\boxed{q^+ = q^0 + q^3 = 0}$$

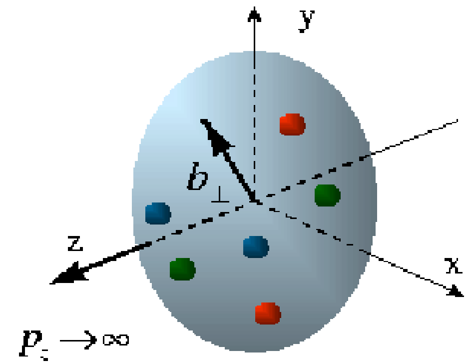
quark transverse charge densities in the nucleon

light-front

$q^+ = q^0 + q^3 = 0$

$Q^2 \equiv \vec{q}_\perp^{\,2}$

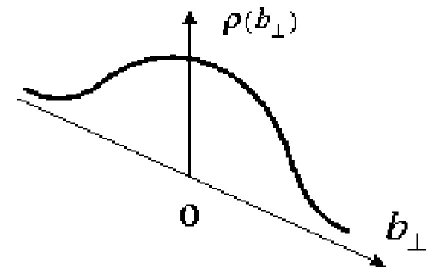




photon only couples to forward moving quarks

quark charge density operator

$$J^+ \equiv J^0 + J^3 = \bar{q}\gamma^+ q = 2\, q_+^\dagger\, q_+, \quad \text{with} \quad q_+ \equiv \frac{1}{4}\gamma^-\gamma^+ q$$



★ longitudinally polarized nucleon

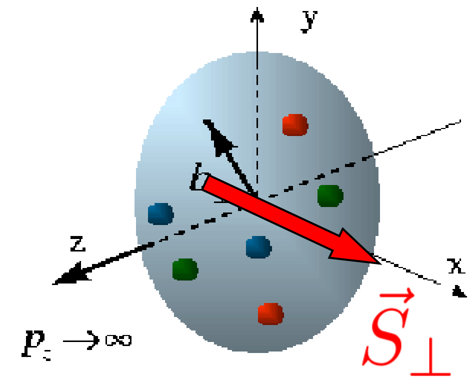
$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\, \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, \lambda\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1(Q^2)$$

G.A. Miller, PRL 99 (2007)

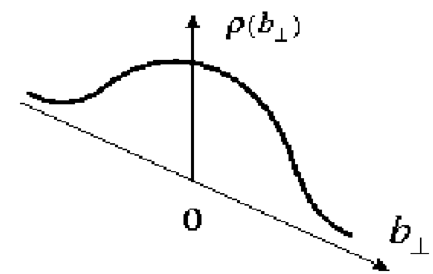
quark transverse charge densities in the nucleon

☆ transversely polarized nucleon



transverse spin $\vec{S}_\perp = \cos\phi_S \, \hat{e}_x + \sin\phi_S \, \hat{e}_y$

e.g. along x-axis : $\phi_S = 0$



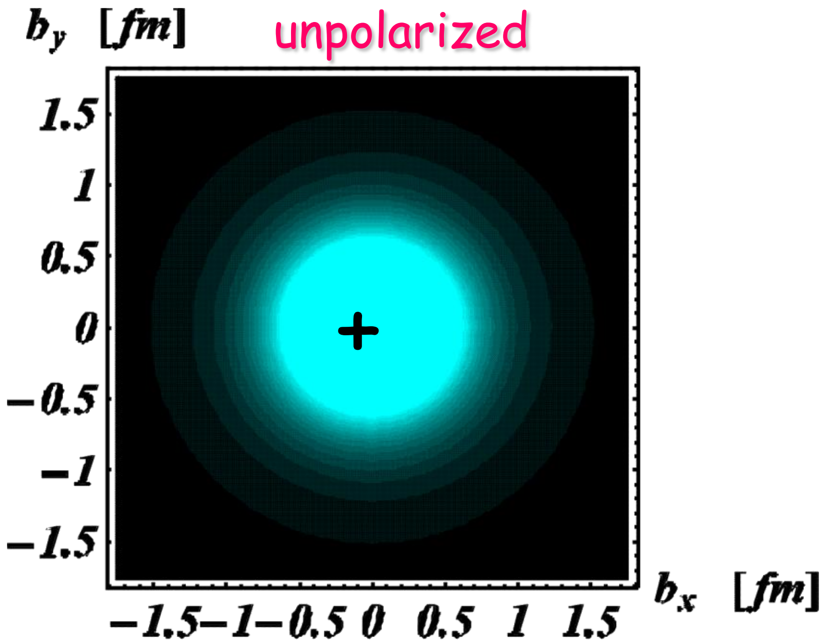
$$\vec{b} = b \, (\cos\phi_b \, \hat{e}_x + \sin\phi_b \, \hat{e}_y)$$

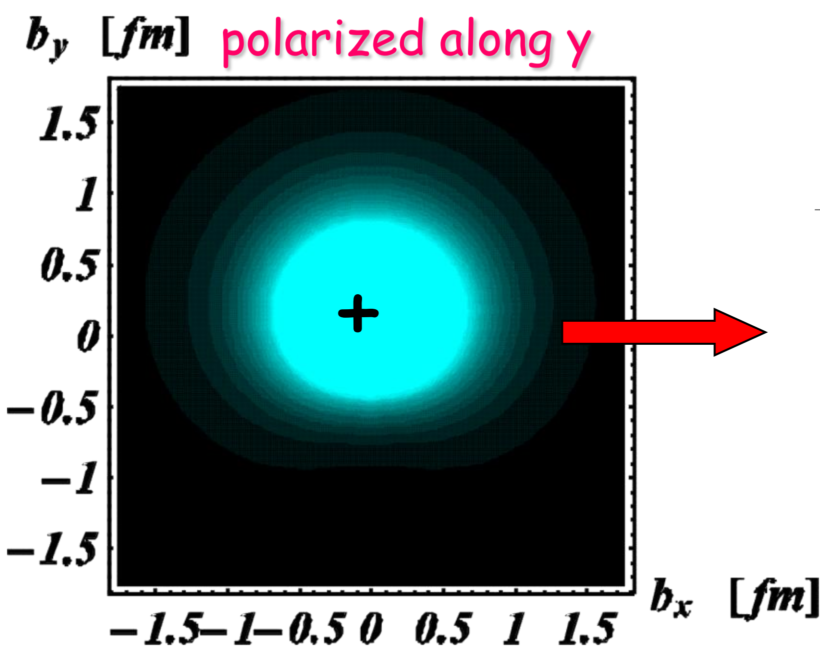
$$\begin{aligned} \rho_T^N(\vec{b}) &\equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} \, e^{-i\,\vec{q}_\perp \cdot \vec{b}} \, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle \\ &= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M_N} J_1(b\,Q) F_2(Q^2) \end{aligned}$$

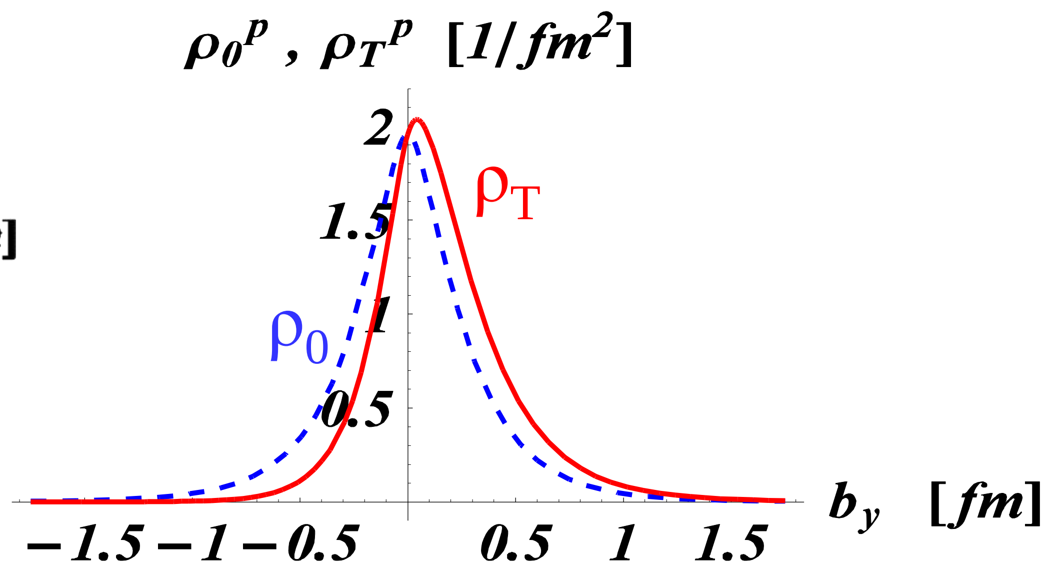
dipole field pattern

Carlson, Vanderhaeghen, PRL 100 (2008)

empirical quark transverse densities in proton





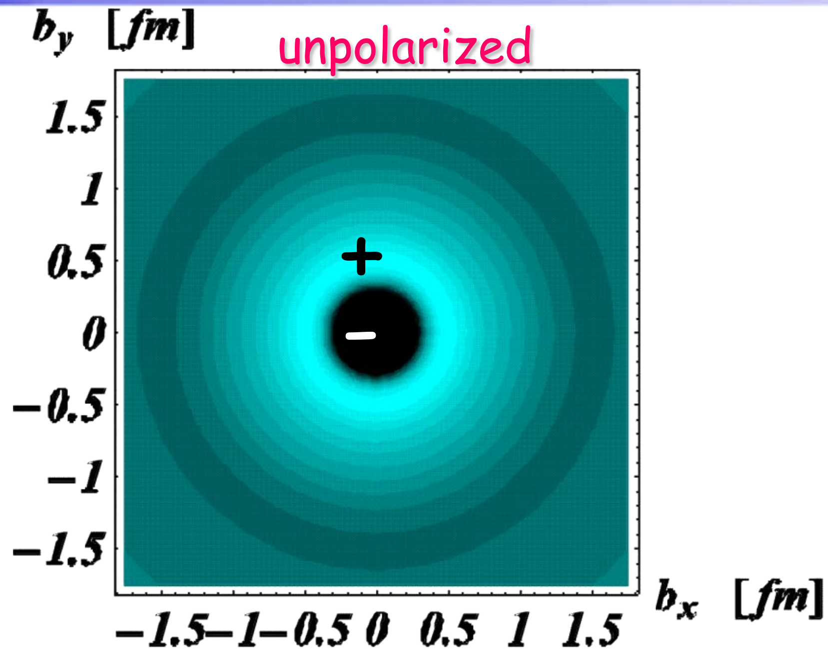


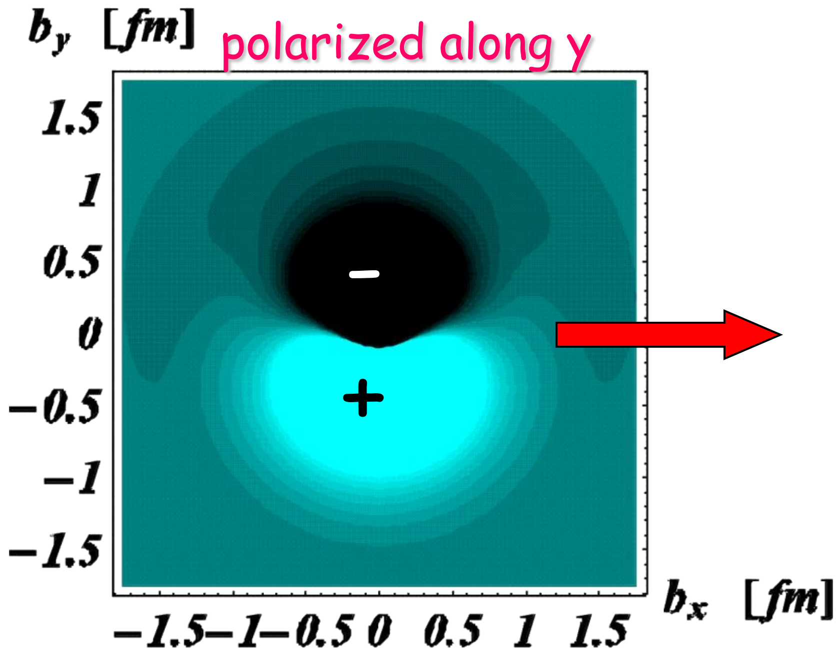
induced **EDM** : $d_y = F_{2p}(0) \,.\, e \,/\, (2\, M_N)$

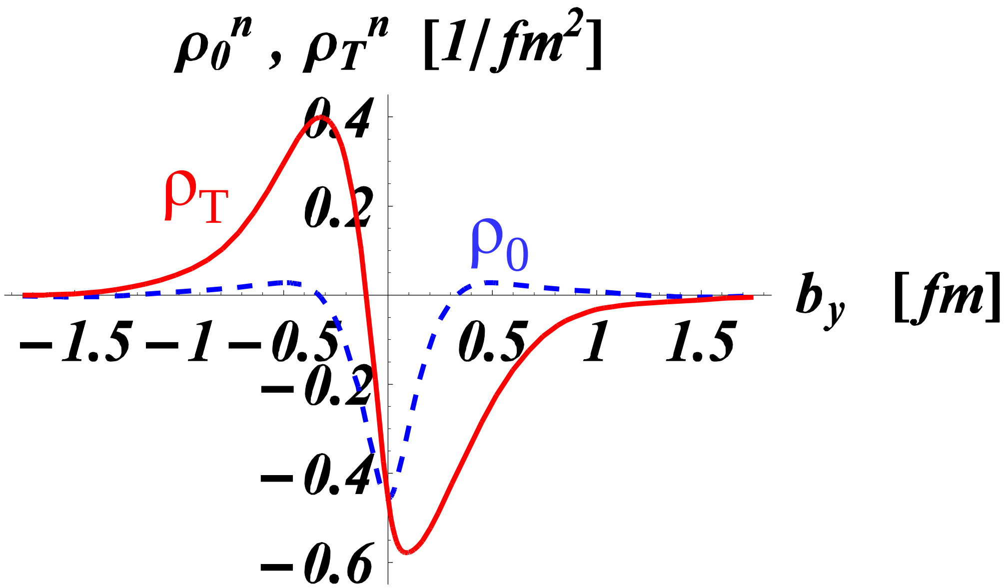
data : **Arrington, Melnitchouk, Tjon** (2007)

densities : **Miller** (2007); **Carlson, Vdh** (2007)

empirical quark transverse densities in neutron







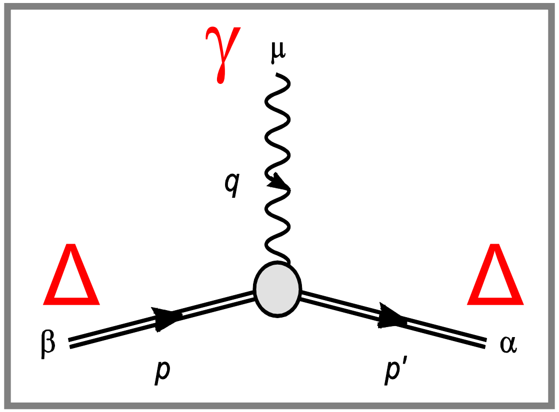
induced EDM : $d_y = F_{2n}(0) . e / (2 M_N)$

data: Bradford, Bodek, Budd, Arrington (2006)

densities : Miller (2007); Carlson, Vdh (2007)

Quark transverse densities in $\Delta(1232)$

$\gamma^* \Delta\Delta$ vertex



$$\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda)\rangle$$
$$= -\bar{u}_\alpha(p',\lambda') \left\{ \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \gamma^\mu \right.$$
$$\left. + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M_\Delta)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M_\Delta} \right\} u_\beta(p,\lambda)$$

multipole transitions

$$G_{E0} = (F_1^* - \tau F_2^*) + \frac{2}{3}\tau G_{E2}$$
electric charge $\quad e_\Delta = G_{E0}(0)$

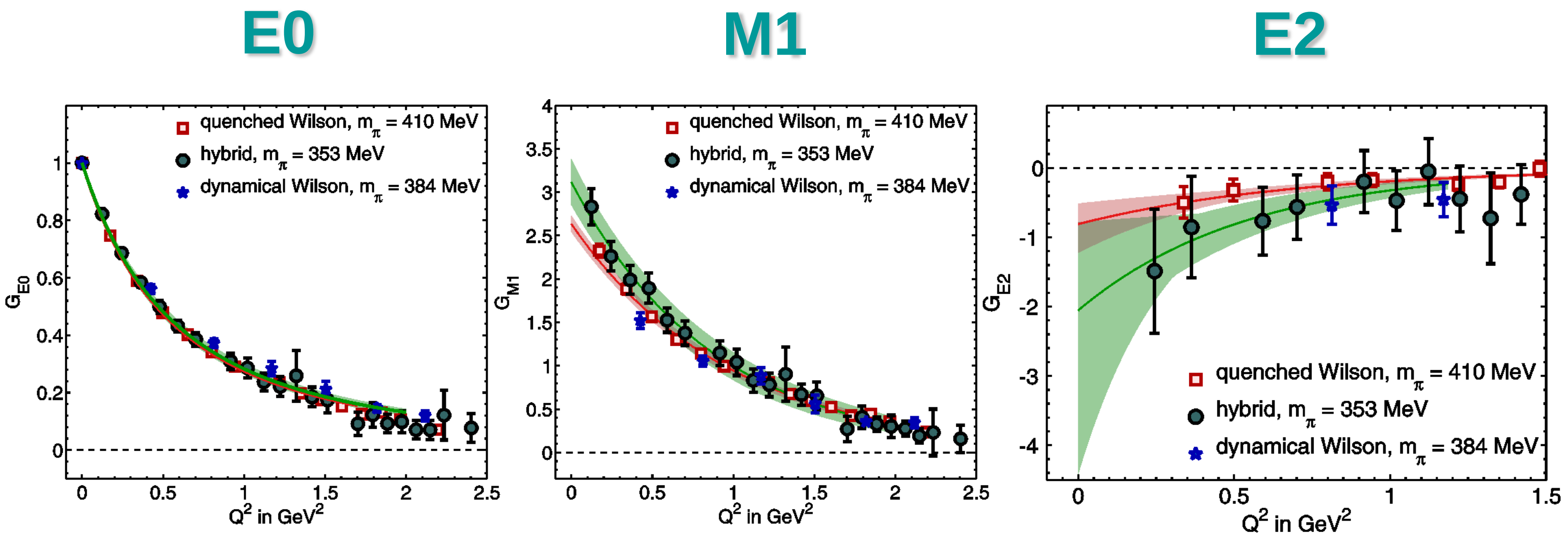
$$G_{E2} = (F_1^* - \tau F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* - \tau F_4^*)$$
charge quadrupole $\quad Q_\Delta = \frac{e}{M_\Delta^2} G_{E2}(0)$

$$G_{M1} = (F_1^* + F_2^*) + \frac{4}{5}\tau G_{M3}$$
magnetic dipole $\quad \mu_\Delta = \frac{e}{2M_\Delta} G_{M1}(0)$

$$G_{M3} = (F_1^* + F_2^*) - \frac{1}{2}(1+\tau)\,(F_3^* + F_4^*)$$
magnetic octupole $\quad O_\Delta = \frac{e}{2M_\Delta^3} G_{M3}(0)$

these elastic form factors cannot be measured !

full QCD lattice calculations



lattice analysis :

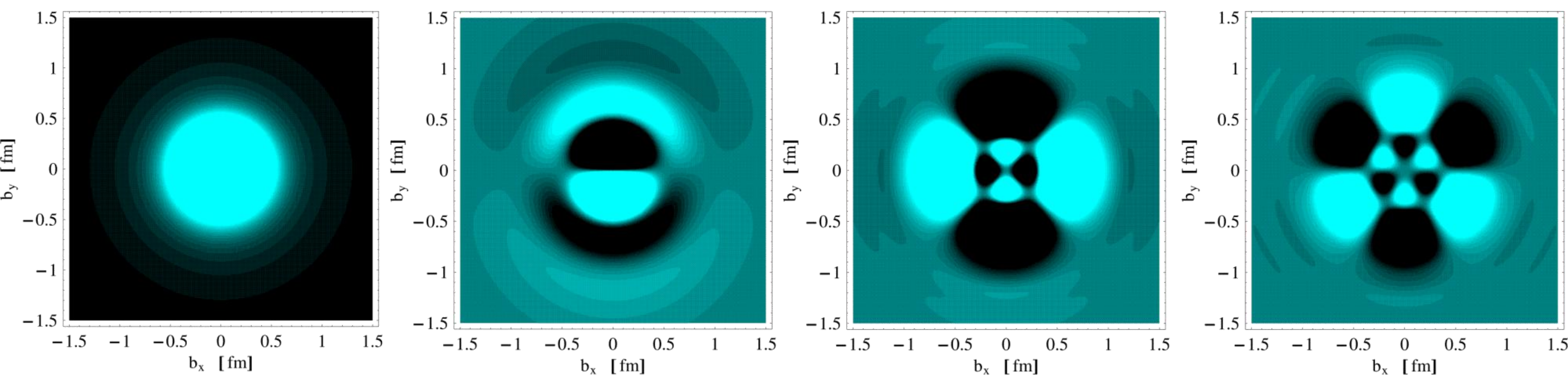
Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tspalis, Vanderhaeghen,

PRD 79, 014507 (2009)

transversely polarized Δ(1232)

$$\rho^{\Delta}_{T\,s_\perp}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+}\langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp \,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp\rangle$$

$$\begin{aligned}
\rho^{\Delta}_{T\,\frac{3}{2}}(\vec{b}) = \int_0^{+\infty} \frac{dQ}{2\pi}\, Q \quad \Big[\quad & J_0(Qb)\,\frac{1}{4}\left(A_{\frac{3}{2}\frac{3}{2}} + 3A_{\frac{1}{2}\frac{1}{2}}\right) && \text{monopole} \\
- \quad & \sin(\phi_b - \phi_S)\, J_1(Qb)\,\frac{1}{4}\left(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} + 3A_{\frac{1}{2}-\frac{1}{2}}\right) && \text{dipole} \\
- \quad & \cos[2(\phi_b - \phi_S)]\, J_2(Qb)\,\frac{\sqrt{3}}{2}A_{\frac{3}{2}-\frac{1}{2}} && \text{quadrupole} \\
+ \quad & \sin[3(\phi_b - \phi_S)]\, J_3(Qb)\,\frac{1}{4}A_{\frac{3}{2}-\frac{3}{2}} \Big] && \text{octupole}
\end{aligned}$$



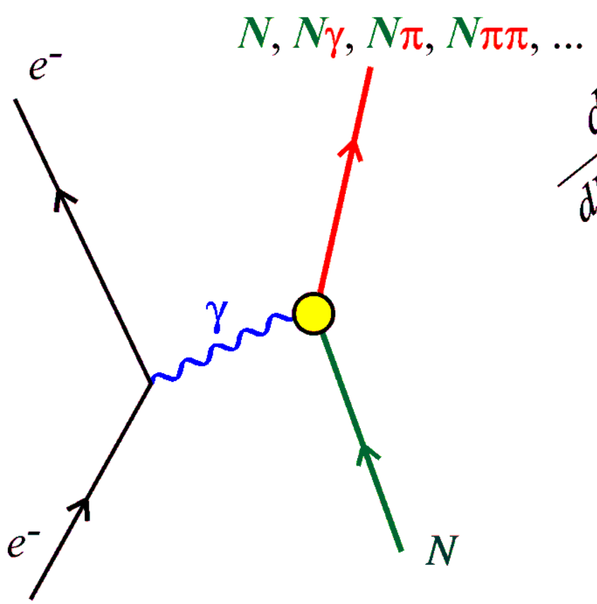
transverse transition charge densities

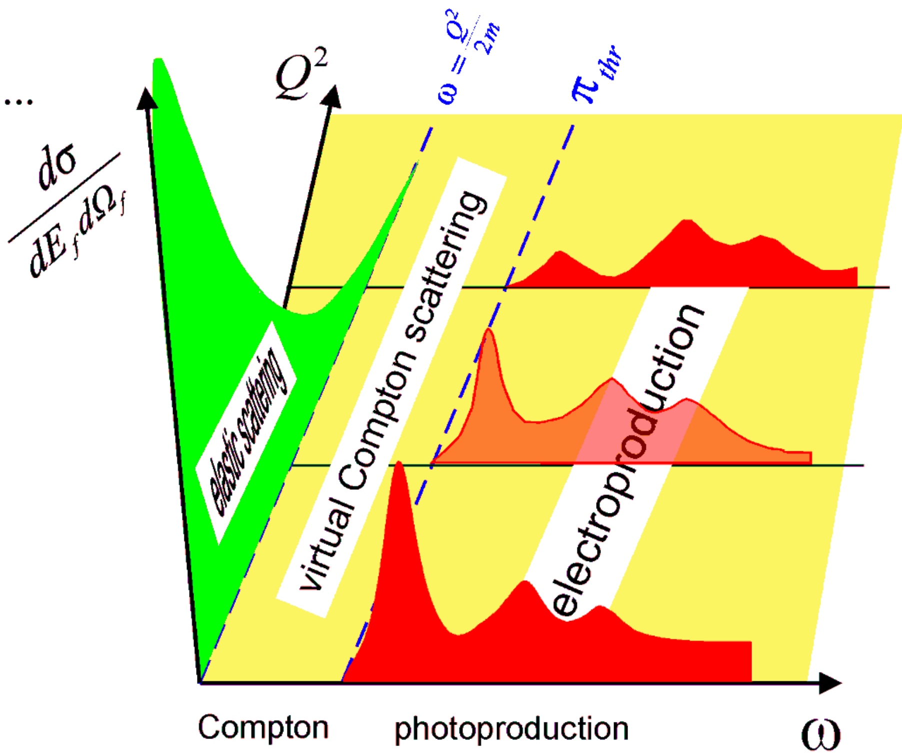
for

N -> Δ (1232), P_{11}(1440),

S_{11}(1535), D_{13}(1520)

transition form factors from electroproduction of π, η, ππ, …





in general:

transition form factors can only be obtained by

- partial wave analysis, e.g. MAID, JLab

  separate S_{11}, P_{11}, P_{33}, D_{13}, F_{15}, etc from angular distributions

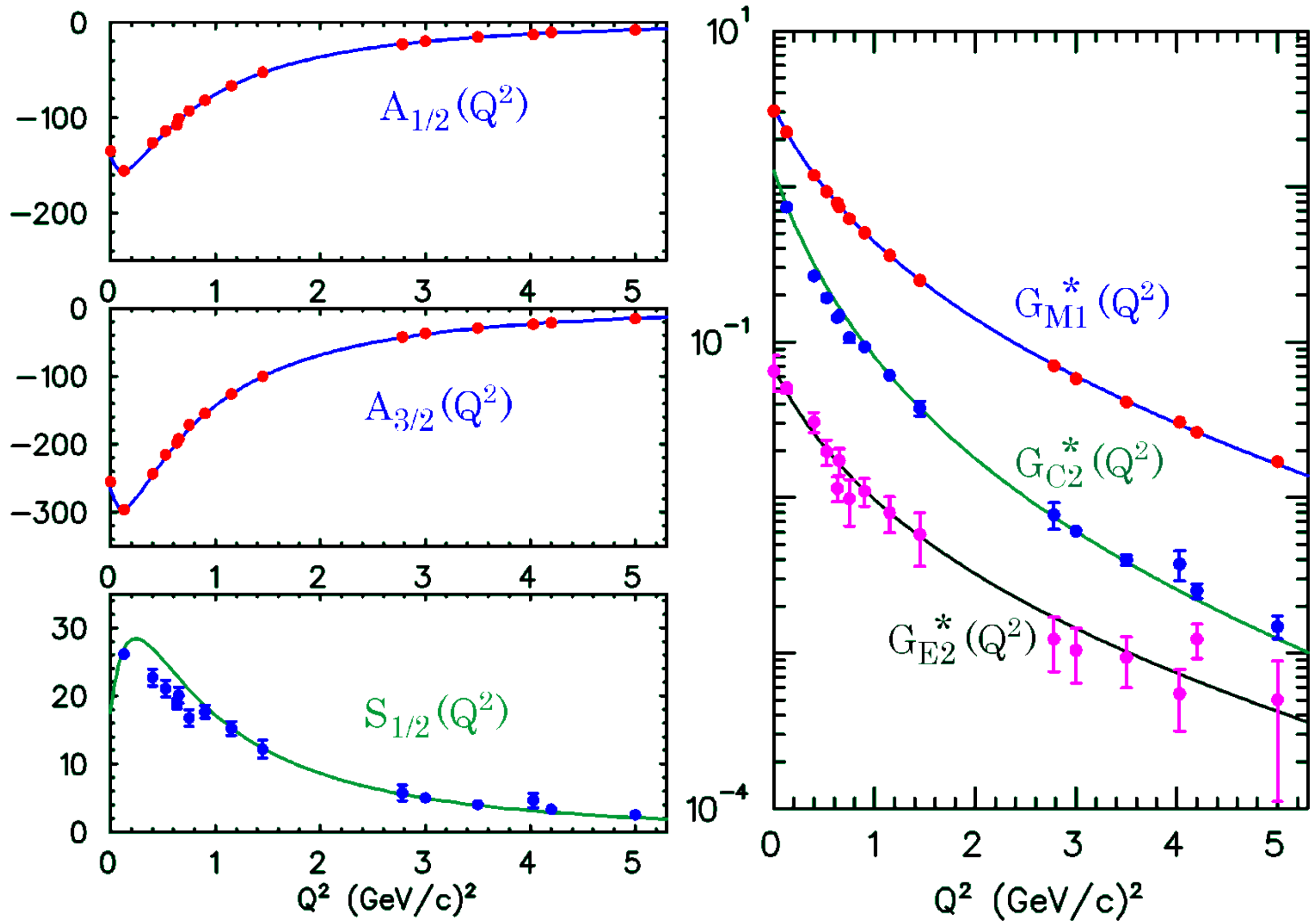
- and background / resonance separation

  separate bg and res parts in each partial wave

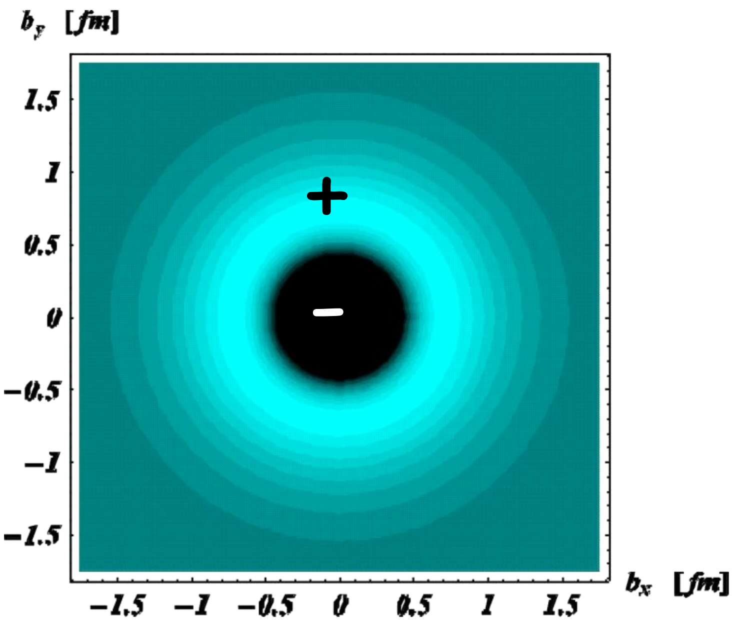
MAID results are available for the dominant resonances of the proton

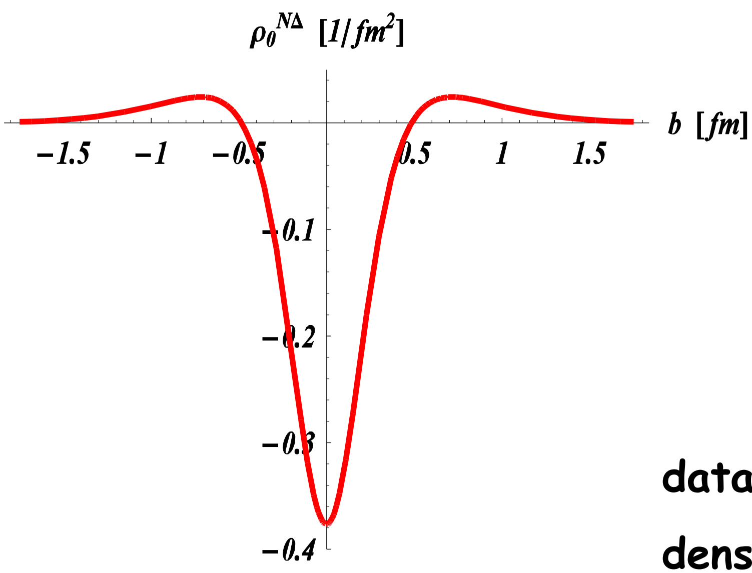
	isospin	spin	form factors
$P_{33}(1232)$	3/2	3/2	G_E, G_M, G_C
$P_{11}(1440)$	1/2	1/2	G_M, G_C
$D_{13}(1520)$	1/2	3/2	G_E, G_M, G_C
$S_{11}(1535)$	1/2	1/2	G_E, G_C
$F_{15}(1680)$	1/2	5/2	G_E, G_M, G_C

magnetic (M1), electric (E2) & Coulomb (C2) N -> Δ transition form factors from MAID analysis



empirical transverse transition densities for N -> Δ excitation





$$\langle P^+, \frac{\vec{q}_\perp}{2}, \lambda_\Delta | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda_N \rangle$$
$$= (2P^+) e^{i(\lambda_N - \lambda_\Delta)\phi_q} \, G^+_{\lambda_\Delta \, \lambda_N}(Q^2)$$

$$\boxed{\rho_0^{N\Delta}(b) = \int_0^\infty \frac{dQ}{2\pi} Q \, J_0(b\,Q) \, G^+_{+\frac{1}{2}\,+\frac{1}{2}}(Q^2)}$$

combination of **M1**, **E2**, **C2** FFs

data : MAID 2007

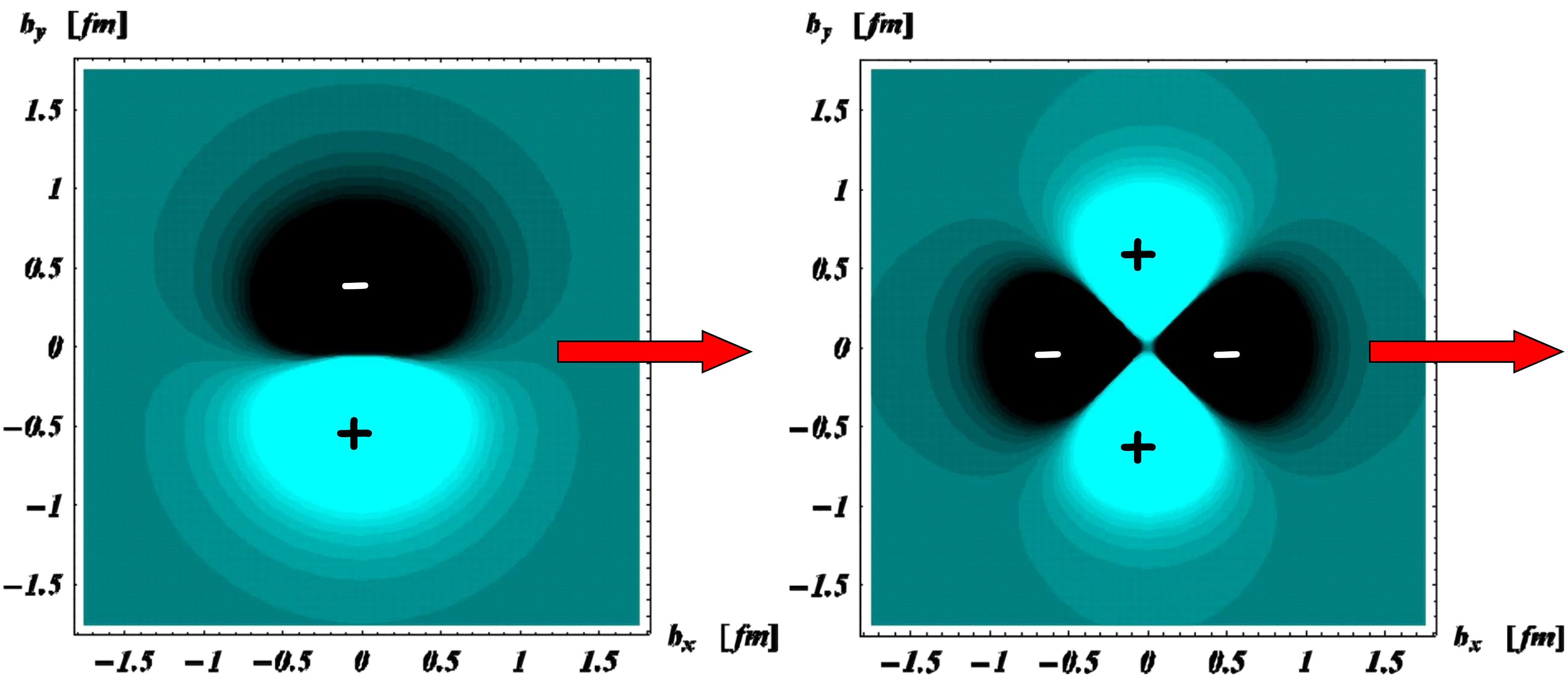
densities : Carlson, Vdh (2007)

$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2}\,|\, J^+(0)\,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2}\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\,\frac{Q}{2}\,\Big\{ J_0(b\,Q)\, G^+_{+\frac{1}{2}\,+\frac{1}{2}} \longrightarrow \text{monopole}$$

$$- \sin(\phi_b - \phi_S)\, J_1(b\,Q) \left[\sqrt{3} G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}}\right] \longrightarrow \text{dipole}$$

$$- \cos 2(\phi_b - \phi_S)\, J_2(b\,Q)\sqrt{3}\, G^+_{+\frac{3}{2}\,-\frac{1}{2}} \Big\} \longrightarrow \text{quadrupole}$$

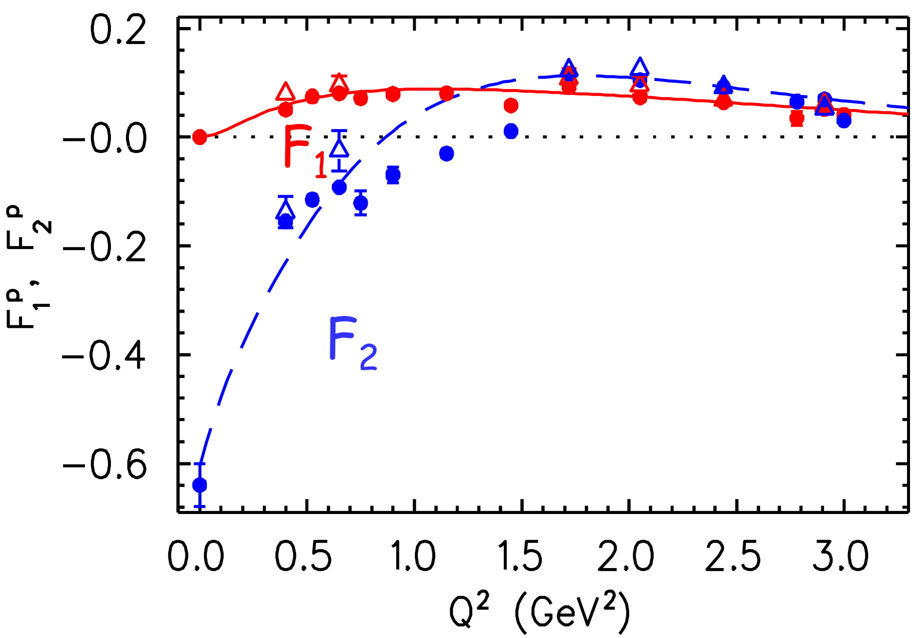


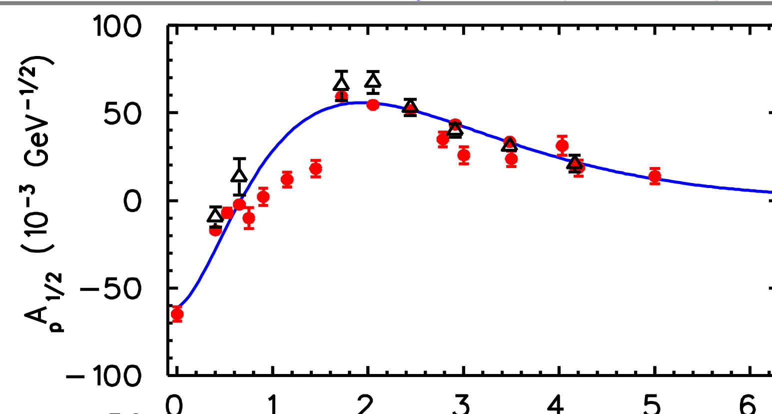
empirical e.m. transition FFs for N -> N*(1440) excitation

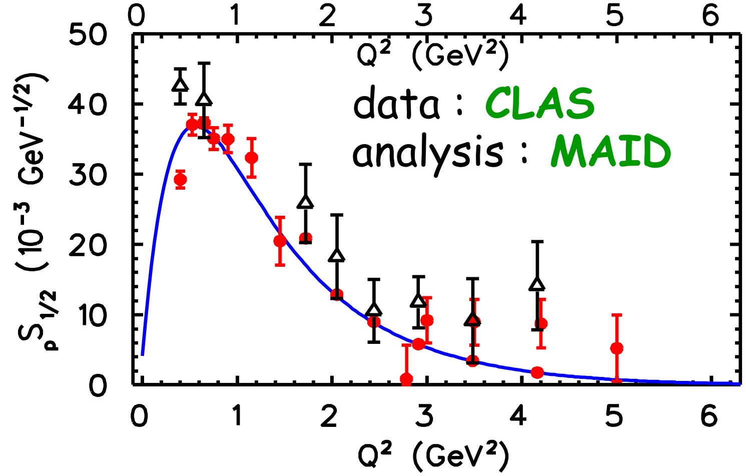
L.T., M. Vanderhaeghen, Phys. Lett. B 672 (2009) 344

$$\langle N^*(p',\lambda')\,|\,J^\mu(0)\,|\,N(p,\lambda)\rangle$$
$$= \bar{u}(p',\lambda')\left\{F_1^{NN^*}(Q^2)\left(\gamma^\mu - \gamma\cdot q\,\frac{q^\mu}{q^2}\right)\right.$$
$$\left. + F_2^{NN^*}(Q^2)\,\frac{i\sigma^{\mu\nu}q_\nu}{(M^*+M_N)}\right\}u(p,\lambda)$$

$$A_{1/2} = e\,\frac{Q_-}{\sqrt{K}\,(4M_N M^*)^{1/2}}\left\{F_1^{NN^*} + F_2^{NN^*}\right\}$$
$$S_{1/2} = e\,\frac{Q_-}{\sqrt{2K}\,(4M_N M^*)^{1/2}}\left(\frac{Q_+Q_-}{2M^*}\right)\frac{(M^*+M_N)}{Q^2}\cdot\left\{F_1^{NN^*} - \frac{Q^2}{(M^*+M_N)^2}F_2^{NN^*}\right\}$$



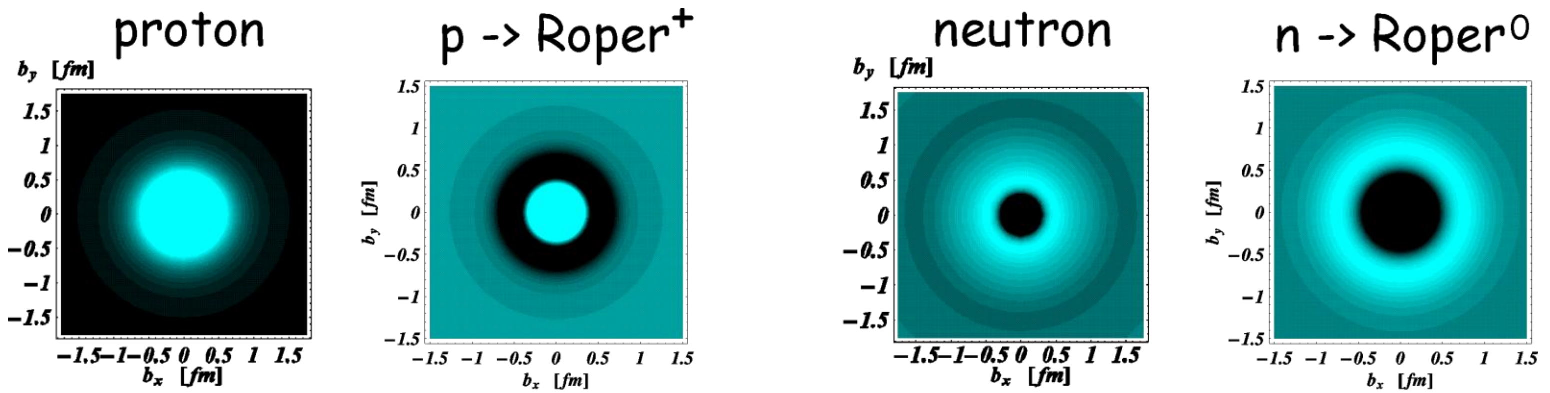




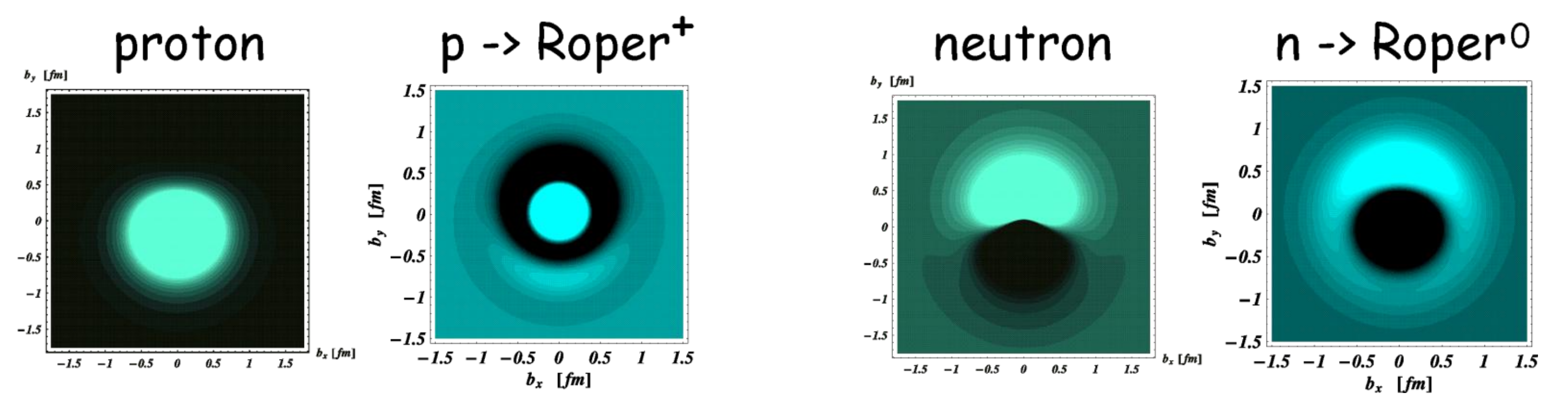
transverse transition densities

for the Nucleon and N -> Roper excitation

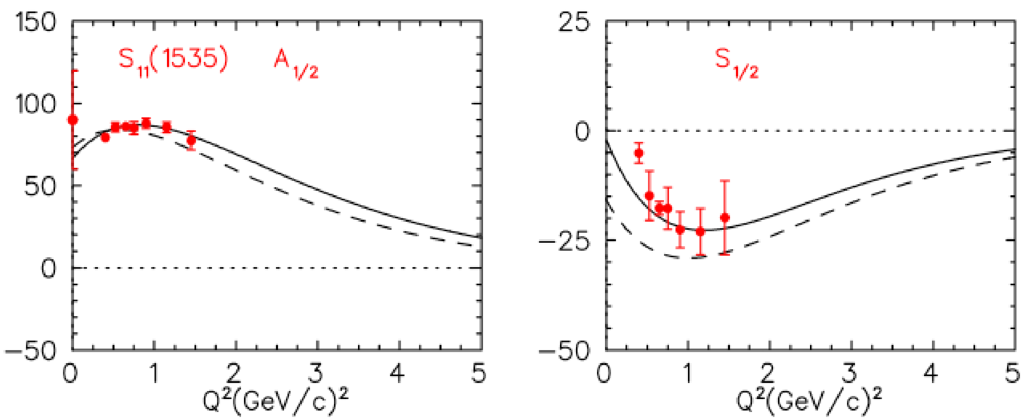
unpolarized: $\rho_0(b) = \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q)\, F_1(Q^2)$



polarized along $\hat{x}$: $\rho_T(\vec{b}) = \rho_0(Q^2) + \frac{b_y}{b} \int_0^\infty \frac{dQ}{2\pi} \tau(Q^2)\, J_1(b\,Q)\, F_2(Q^2)$



empirical transition FFs for p -> S_{11}(1535)

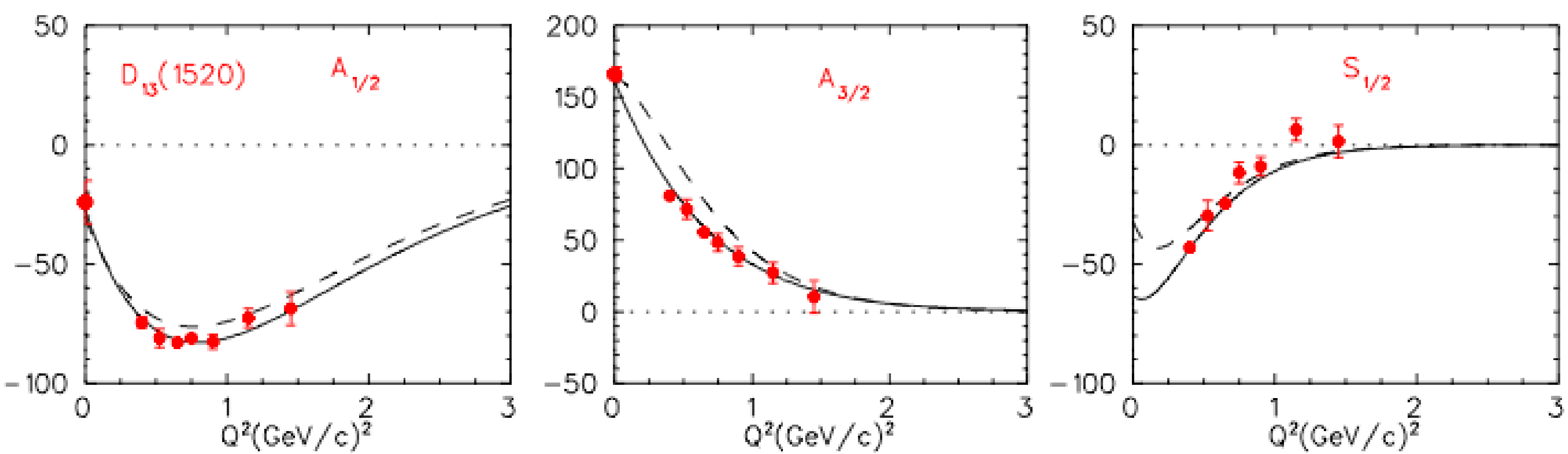


data : CLAS

analysis : MAID

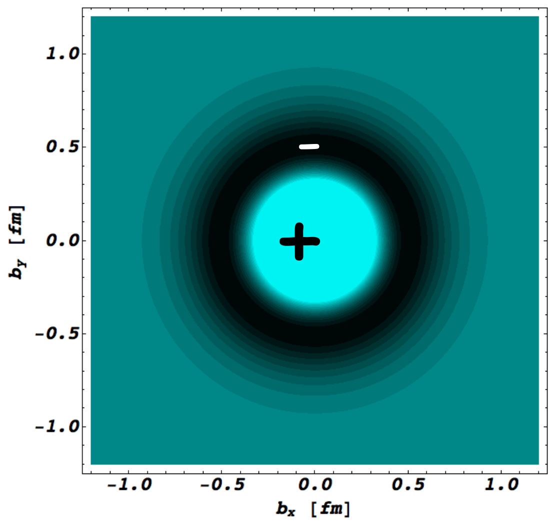
2007
2003

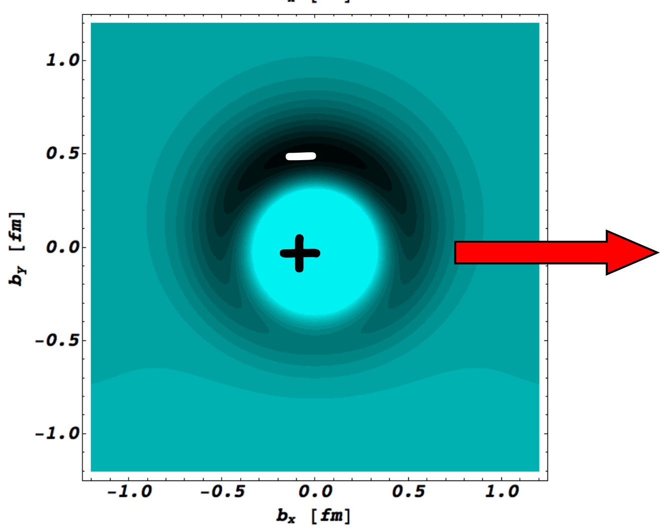
and for p -> D13(1520) excitations



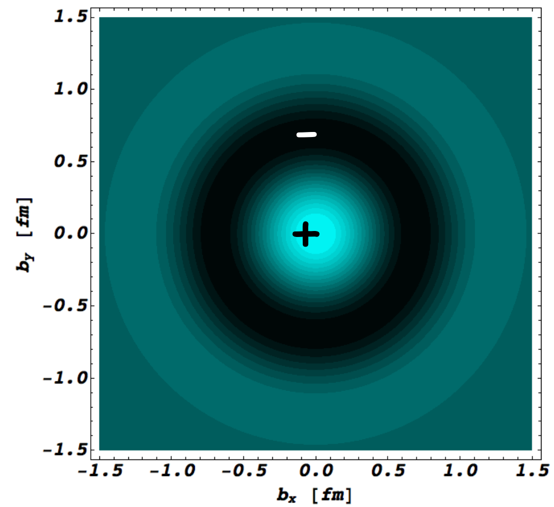
transverse transition densities for

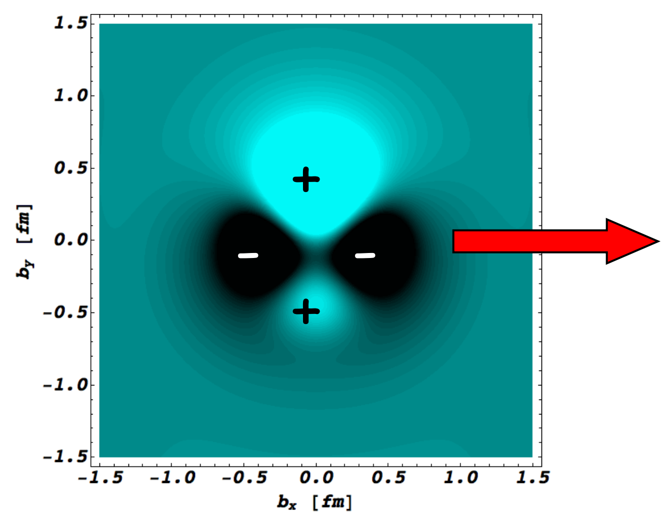
p -> S_{11}(1535)





p -> D_{13}(1520)





Summary and Conclusions

Transverse charge densities
give us a new view of the nucleon and
nucleon resonances

with N->N* transition form factors
obtained from the MAID analysis
with mosty JLab data
we can visualize quark transition charge densities
for the Delta, Roper, S11 and D13 resonances

furthermore, with elastic N* form factors
obtained from Lattice QCD
we can even visualize quark charge densities
for resonances themselves, e.g. Delta
and soon probably also for Roper and S11